MATH 147 Review: Partial Derivatives

Facts to Know

Notation

$$f_x(x,y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x,y) = \frac{\partial z}{\partial x} = f_1 = D_1 f = D_x f$$

$$f_y(x,y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y} f(x,y) = \frac{\partial z}{\partial y} = f_2 = D_2 f = D_y f$$

Rule for finding partial derivatives of $z = f(x,y)$

f_x x is the only variable (y is a constant)

f_y y is the only variable (x is constant)

Examples

1. Let $f(x,y) = x^3 + x^2y^3 - 2y^2$. Find f_x and f_y and f_{xy} and f_{yx}.

$$f_x = \frac{\partial}{\partial x}(x^3 + x^2y^3 - 2y^2) = \frac{\partial}{\partial x}(x^3) + \frac{\partial}{\partial x}(x^2y^3) + \frac{\partial}{\partial x}(-2y^2)$$

$$= 3x^2 + y^3(2x) + 0$$

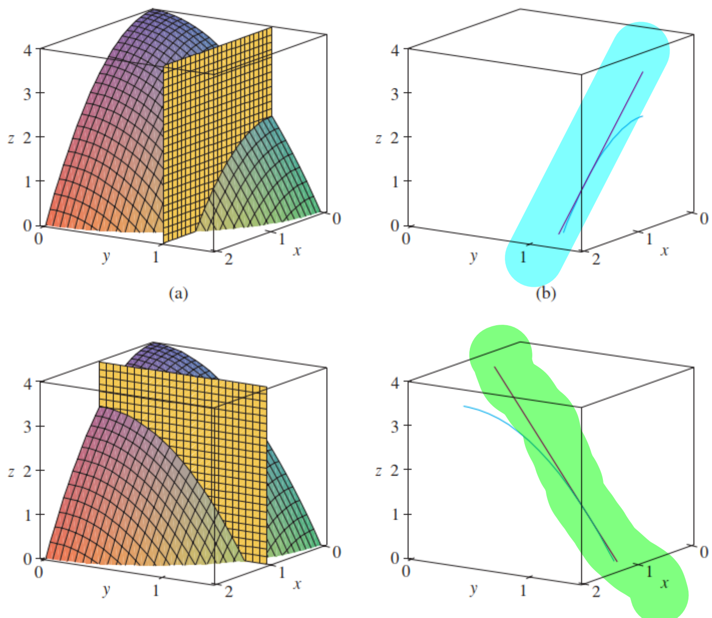
$$= 3x^2 + 2xy^3$$

$$f_{xy} = (f_x)_y = (3x^2 + 2xy^3)_y = 0 + 2x(3y^2)$$

$$f_y = \frac{\partial}{\partial y}(x^3 + x^2y^3 - 2y^2) = 0 + x^2(3y^2) - 4y$$

$$f_{yx} = (f_y)_x = (3x^2y^2 - 4y)_x = 6xy^2$$

2. Let $f(x, y) = 4 - x^2 - 2y^2$. Find $f_x(1, 1)$ and $f_y(1, 1)$.



$f_x = 0 + (-2x) + 0$

$f_y = 0 + 0 + -4y$

$f_x(1,1) = -2(1) = -2$

$f_y(1,1) = -4(1) = -4$